



# Dealing with rare events in Cochrane reviews

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# Structure of session

- Background
- Rare events in Cochrane reviews
- Statistical approaches for analyzing rare outcomes
  - Standard meta-analytical models and their limitations
  - Alternative meta-analytical models
- Take home messages
- Discussion and questions

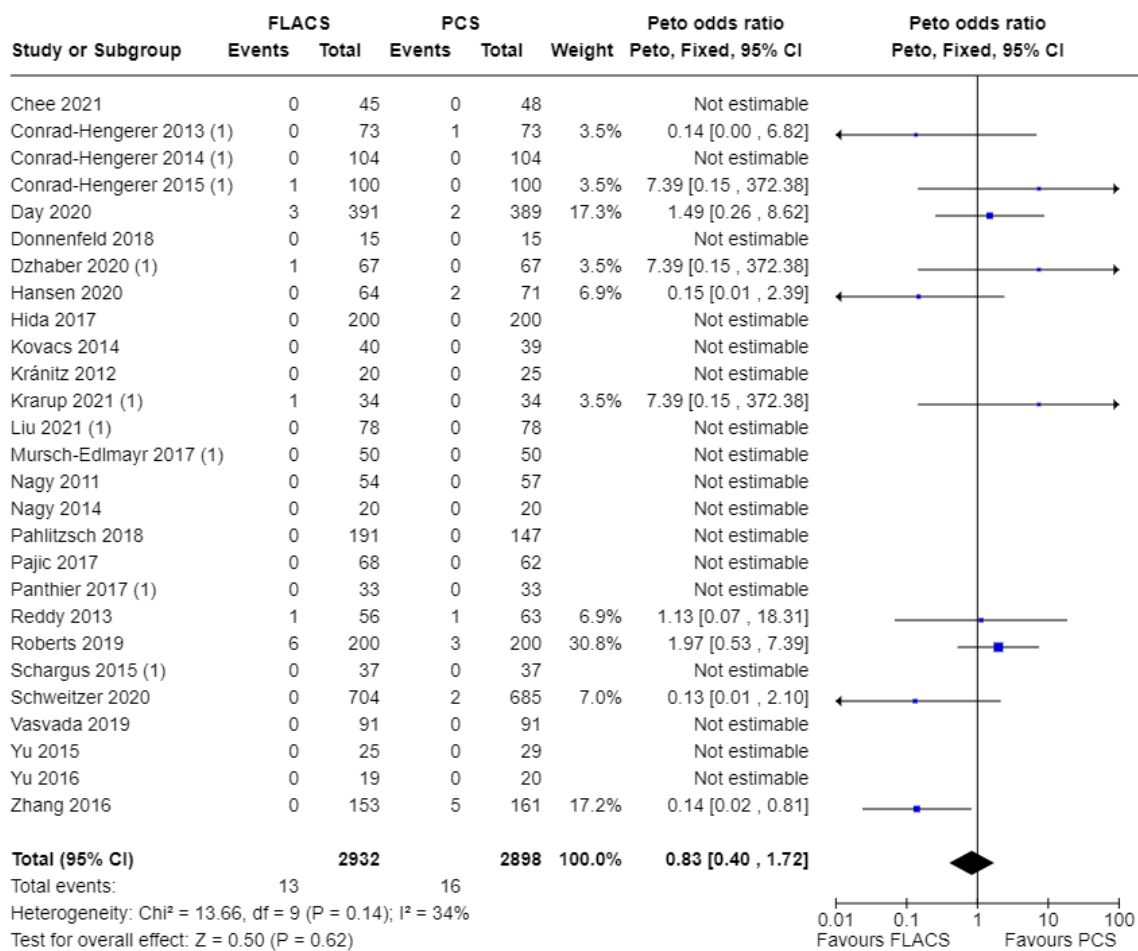


# Background

- No globally accepted definition of what constitutes a rare event
  - Small number of events (even zero) is observed in the studies at hand
  - Outcome risk  $<1\%$  (or, even  $<0.1\%$ )
- Common issue when studying important safety dichotomous outcomes (e.g. different types of adverse effects)
- Meta-analysis (MA) is a powerful tool for synthesizing individual studies (usually underpowered to detect any treatment effects due to rare outcomes) and increase the overall statistical power



# Background



- 27 studies overall
- Maximum number of events per arm: 6 (Roberts 2019)
- 17 ‘double-zero’ studies
- 10 studies contributed to the summary estimate

# Rare events in Cochrane reviews

- In a sample of 480 Cochrane reviews,
  - 35% did an adverse events MA
  - ~30% of the MAs had at least one study with zero events in one arm
  - Most common summary estimates:
    - Peto's odds ratio
    - Mantel-Haenszel odds ratio
    - Mantel-Haenszel risk ratio
    - Risk difference

*Statistical Methods in Medical Research* 2009; **18**: 421–432

## **Meta-analyses of safety data: a comparison of exact versus asymptotic methods**

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University of Alberta Evidence Based Practice Centre, Alberta Research Centre for Health  
Evidence, **Bradley C Johnston** University of Alberta, Department of Medicine and  
**Natasha Wiebe** University of Alberta Clinical Nephrology Research Group

# Rare events in Cochrane reviews

- In a sample of 4,177 rare events MAs included in Cochrane reviews, only 12% had sufficient power ( $\geq 80\%$ ) to detect a relative risk reduction of 50%
- For MAs of rare events, a much larger number of studies was needed to ensure sufficient power



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## ORIGINAL ARTICLE

Many meta-analyses of rare events in the Cochrane Database of Systematic Reviews were underpowered

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# Standard meta-analytical models

- Approaches available in RevMan:
  - Fixed- and random-effects inverse-variance (IV) methods
  - Mantel-Haenszel (MH) method
  - Peto's method



## Fixed- and random-effects IV methods

- Estimate of the intervention effect and its standard error from each study
- A **fixed-effect MA** is valid under an assumption that all effect estimates are estimating the same underlying intervention effect

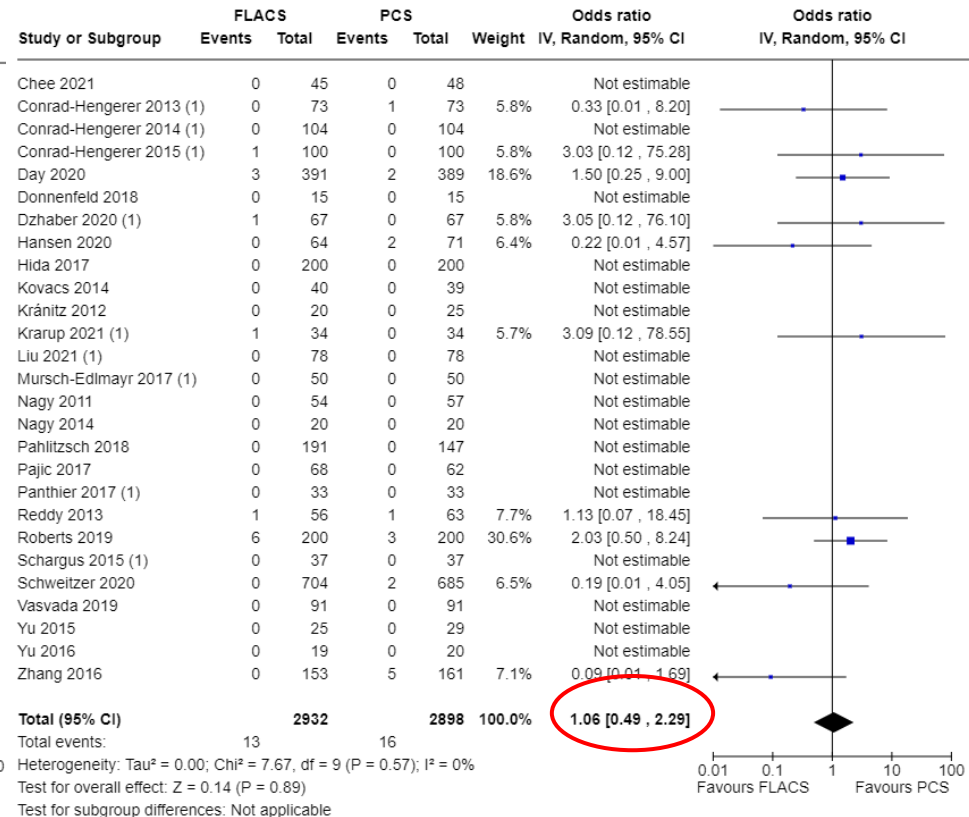
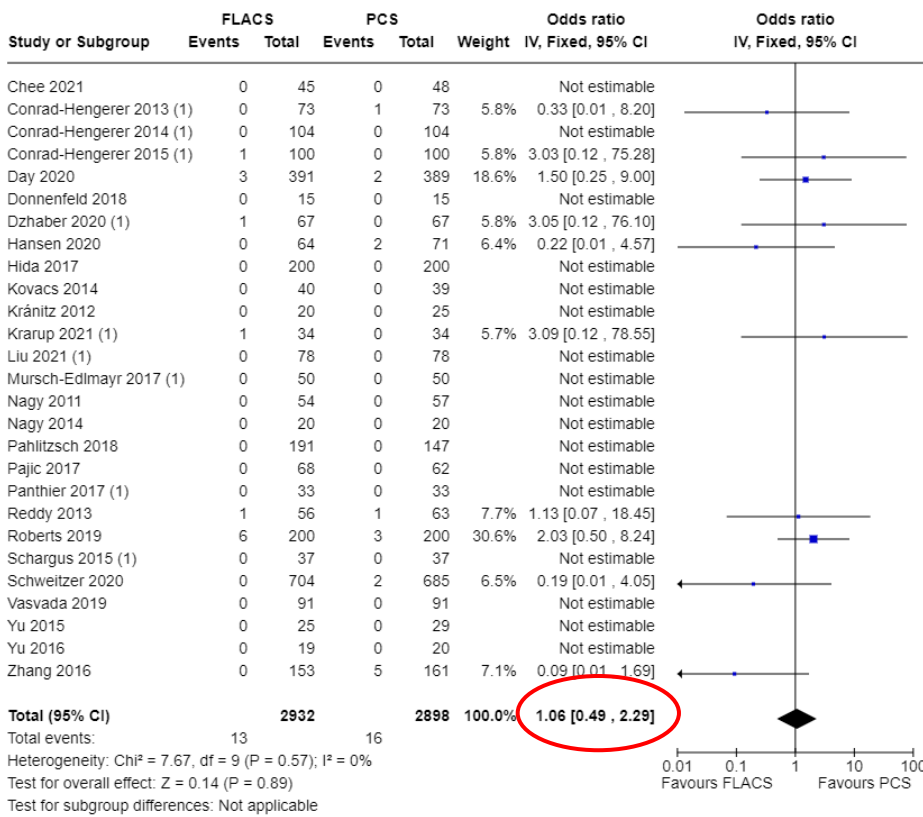
- $$\mu_{IV-FE} = \frac{\sum_{i=1}^k w_i y_i}{\sum_{i=1}^k w_i}, \text{ where } w_i = \frac{1}{SE_i^2}$$

- A **random-effects MA** (DerSimonian & Laird) assumes that the different studies are estimating different, yet related, intervention effects

- $$\mu_{IV-RE} = \frac{\sum_{i=1}^k w_i^* y_i}{\sum_{i=1}^k w_i^*}, \text{ where } w_i^* = \frac{1}{SE_i^2 + \tau^2}$$



# Fixed- and random-effects IV methods



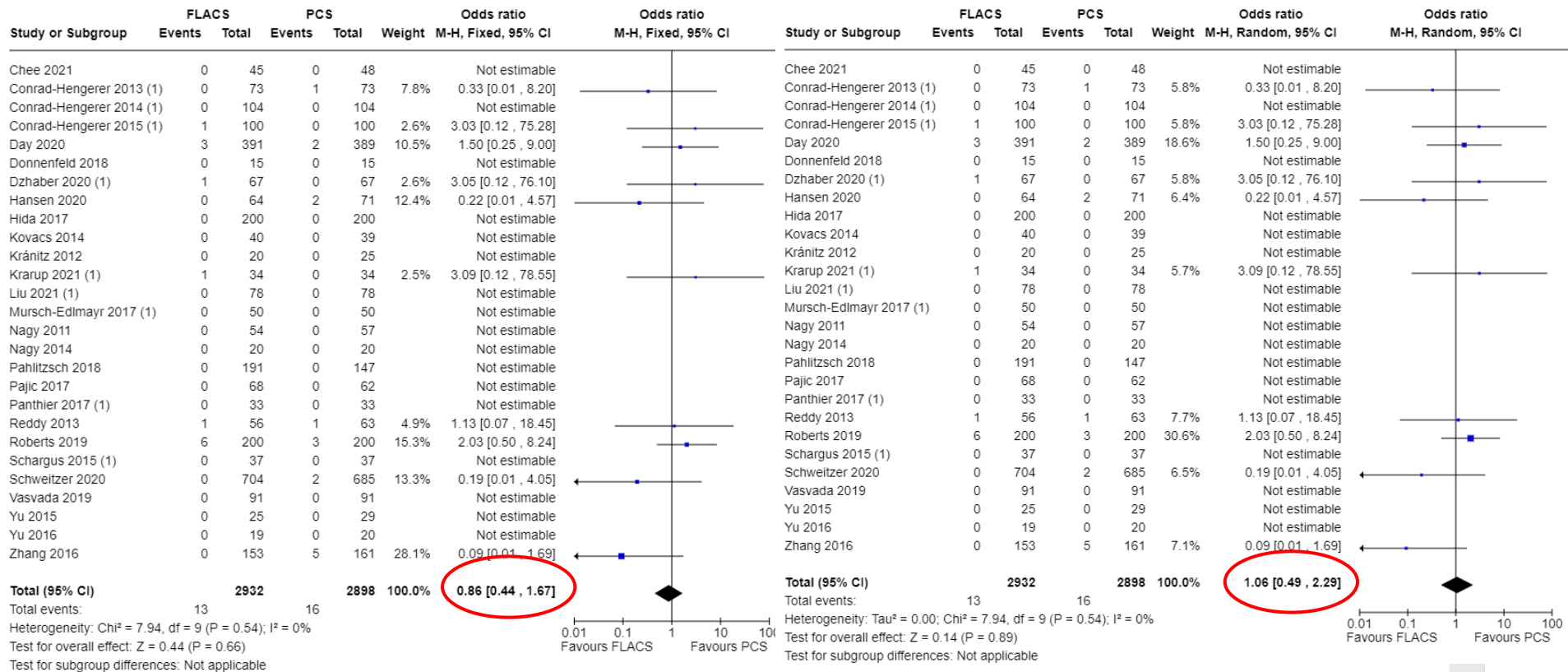
## Mantel-Haenszel (MH) method

- A fixed-effect approach using a different weighting scheme that depends on which effect measure (e.g. risk ratio, odds ratio, risk difference) is being used
- No distributional assumption (i.e. a non-parametric approach)

	No. of events	No. of non- events	Total
Intervention	a	b	a+b
Control	c	d	c+d
Total	a+c	b+d	n=a+b+c+d

- $OR_i = \frac{a/b}{c/d} = \frac{ad}{bc}$ ,  $OR_{MH} = \frac{\sum_{i=1}^k \frac{a_i d_i}{n_i}}{\sum_{i=1}^k \frac{b_i c_i}{n_i}}$

# Mantel-Haenszel (MH) method



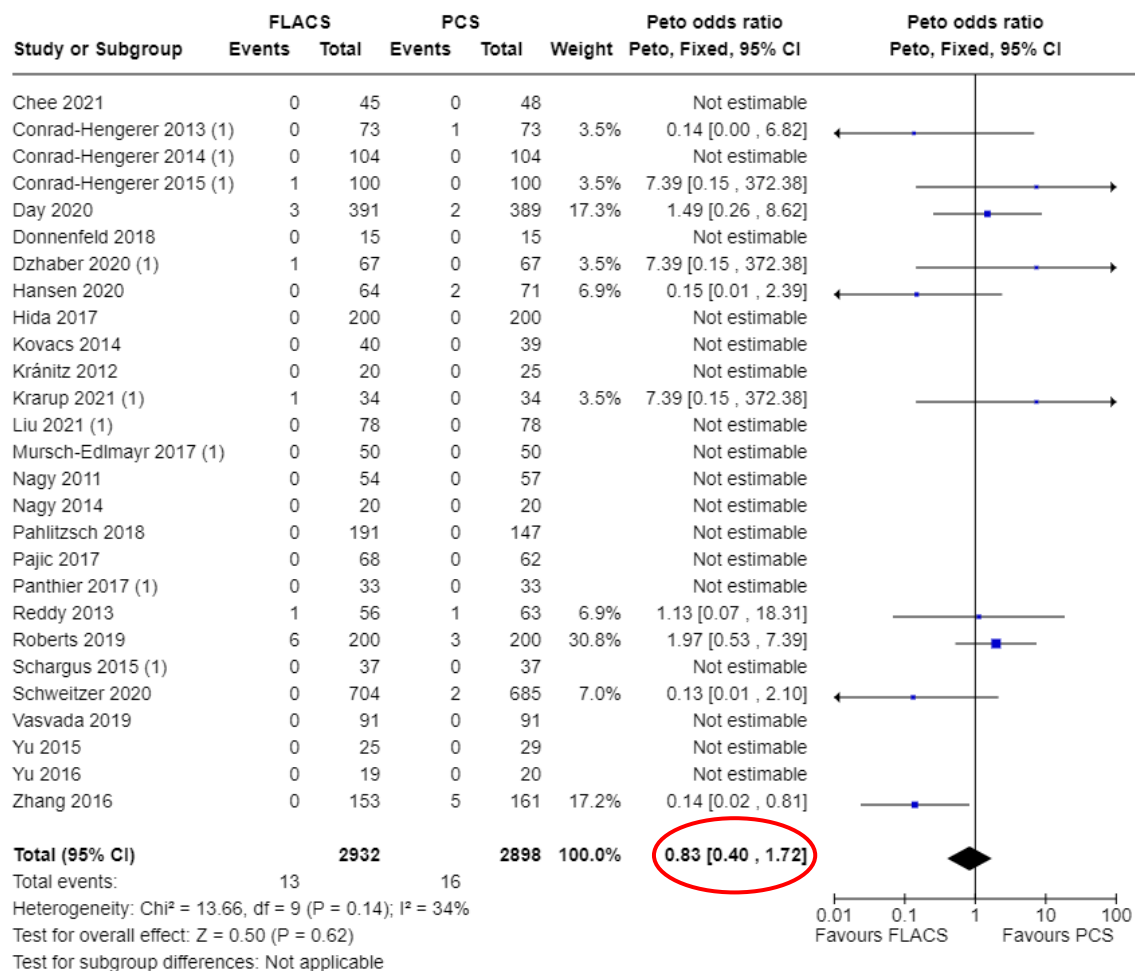
## Peto's method

- A fixed-effect IV approach to combine odds ratios only
- Again, no distributional assumption

	No. of events	No. of non- events	Total
Intervention	a	b	a+b
Control	c	d	c+d
Total	a+c	b+d	n=a+b+c+d

- $\mu_{Peto} = \exp\left(\frac{\sum_{i=1}^k (O_i - E_i)}{\sum_{i=1}^k V_i}\right)$ , where  $O_i = a$ ,  $E_i = \frac{(a+b)(a+c)}{n}$ ,  
 $V_i = \frac{(a+b)(c+d)(a+c)(b+d)}{n}$

# Peto's method



## Summary of methods

Method	OR (95% CI)
Fixed-effect IV	1.06 (0.49-2.29)
Random-effects IV (DerSimonian & Laird)	1.06 (0.49-2.29)
Fixed-effect MH	0.86 (0.44-1.67)
Random-effects MH	1.06 (0.49-2.29)
Peto	0.83 (0.40-1.72)
<i>CI; confidence interval, OR; odds ratio</i>	

Poll: Which method would you choose for this review?

# Limitations of standard meta-analytical models

- **In general, IV methods are not appropriate for MAs of rare events**
  - Use of a normal approximation of the true binomial likelihood (the ‘large sample approximation’) which does not work well when event rates are low
  - The estimation of the variance of random effects (heterogeneity) may be biased, which may lead to spuriously narrow confidence intervals





# Limitations of standard meta-analytical models

- In simulations studies, Peto method gave the least biased summary estimate and best confidence interval coverage if:
  - There was no substantial imbalance between intervention and control group sizes
  - Treatment effects were not exceptionally large
- Peto is problematic when zero events occur in all arms of all studies

ORIGINAL ARTICLE | VOLUME 91, P129-136, NOVEMBER 2017

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The Yusuf-Peto method was not a robust method for meta-analyses of rare events data from antidepressant trials

Tarang Sharma   • Peter C. Gøtzsche • Oliver Kuss

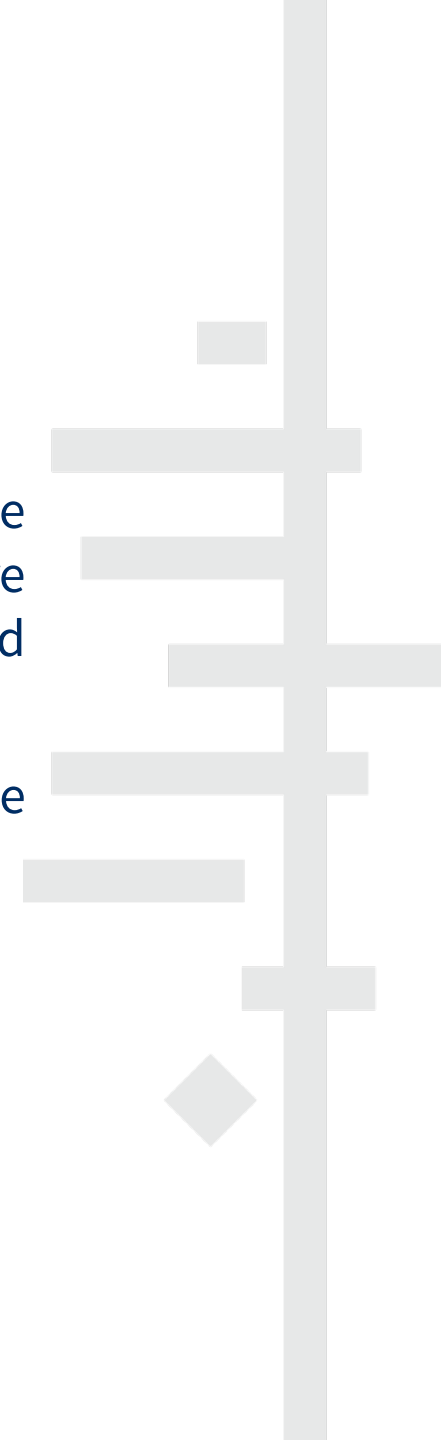
Published: August 09, 2017 • DOI: <https://doi.org/10.1016/j.jclinepi.2017.07.006> •

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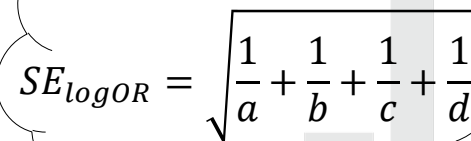
# Limitations of standard meta-analytical models

- In other circumstances (i.e. event risks above 1%, very large effects, and meta-analyses where many studies are substantially imbalanced), the MH fixed-effect method should be preferred
- But MH is also problematic when zero events exist for the same arm across the studies



# Limitations of standard meta-analytical models

- Studies with zero events are excluded since calculation of treatment effects and their corresponding standard errors becomes impossible (it involves division by zero)
- By default, RevMan does a continuity correction (adding 0.5) to studies with zero events *in only one arm*


$$SE_{\log OR} = \sqrt{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}}$$

## Possible remedies

- Continuity correction
- Use of risk differences



## Continuity correction

Method	Non-zero events in both arms (3 studies)	+Zero events in one arm (10 studies)	+Zero events in both arms (27 studies)
Fixed-effect IV	1.70 (0.61-4.74)	1.06 (0.49-2.29)	1.04 (0.57-1.90)
Random- effects IV	1.70 (0.61-4.74)	1.06 (0.49-2.29)	1.04 (0.57-1.90)
Fixed-effect MH	0.83 (0.40-1.72)	0.86 (0.44-1.67)	0.91 (0.53-1.56)
Random-effects MH	1.70 (0.61-4.74)	1.06 (0.49-2.29)	1.04 (0.57-1.90)
Peto	1.68 (0.63-4.52)	0.83 (0.40-1.72)	0.89 (0.50-1.59)

## Continuity correction

- Simulation studies report excessively biased estimates after applying a 0.5 continuity correction, especially in fixed- and random-effects models
- Another method is to use non-fixed corrections (the continuity correction is different for each treatment arm of each study, and is inversely related to the size of the treatment arm) but it has been criticized as well



## Use of risk differences

Method	OR (95% CI) (10 studies)	RD (95% CI) (27 studies)
Fixed-effect IV	1.06 (0.49-2.29)	-0.001 (-0.005-0.002)
Random-effects IV (DerSimonian & Laird)	1.06 (0.49-2.29)	-0.001 (-0.005-0.002)
Fixed-effect MH	0.86 (0.44-1.67)	-0.001 (-0.006-0.004)
Random-effects MH	1.06 (0.49-2.29)	-0.001 (-0.005-0.002)
Peto	0.83 (0.40-1.72)	-

*CI; confidence interval, OR; odds ratio, RD: risk difference*

- In the presence of rare events, risk difference methods have poor statistical properties (too wide intervals and low power), which makes them unsuitable for MA

# Alternative meta-analytical models

- MA models using simple or penalized logistic regression
- Bayesian methods
- MA models using arcsine difference
- Beta-binomial model with correlated responses
- Exact methods based on combining CIs and p-value functions
- Bivariate binomial-normal model
- Non-central hypergeometric model
- Poisson-gamma models



# Alternative meta-analytical models

- Simulation studies show:
  - Simple logistic regression performs similarly with the MH method with no continuity correction
  - In a Bayesian meta-analysis of rare events, the choice of prior distributions is very important as non-informative priors may dominate results
  - Beta-binomial with correlated responses can include studies with zero events in one or both arms, without continuity correction. It has been shown to perform well in various settings, when studies are balanced.





# Alternative meta-analytical models

- Simulation studies show:
  - The use of arcsine difference as an effect measure tackles all problems associated with rare events in meta-analysis. But, difficult to interpret in clinical terms
  - Heterogeneity plays a significant role but this should not be a concern now!



# Take home messages

- In presence of rare events,
  - MA might be the only way to investigate relevant outcomes
  - The IV models should be avoided
    - Assumption of within-studies normality does not hold
    - Biased estimates
  - Continuity correction should not be performed
  - Calculation of risk differences should be avoided
  - Peto's method and fixed-effect MH without continuity correction seem to work well under certain circumstances



## Take home messages

- In presence of rare events,
  - Alternative meta-analytical models have been proposed which incorporate studies with zero events in one or both arms
  - Estimates will be inevitably imprecise
    - Results may be very sensitive to the choice of method used to analyze data
    - Sensitivity analyses should be done using a variety of pre-defined methods
    - Relative effects should be presented along with absolute event rates

## Take home messages

- In presence of rare events,
  - When different methods lead to results with different clinical implications, results should be interpreted with caution. In such cases, results should be considered as hypothesis generating
- Talk with a statistician!



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# Discussion and questions

